Gravitational Lensing as Anisotropic Refraction in an Effective Hyper-Elastic Continuum: A Proof of Concept

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Abstract

Classical refractive models of gravity historically failed to reproduce the full general-relativistic prediction for gravitational light deflection, yielding only half the observed value. This failure has long been cited as evidence that no medium-based mechanism can replace spacetime curvature. We revisit the problem using modern continuum mechanics, modeling the vacuum as an effective hyper-elastic continuum with two independent material responses: an effective density gradient and a stress-dependent softening of the shear modulus via the acousto-elastic effect. When both effects are included, the resulting refractive-index gradient reproduces the full Schwarzschild light-deflection angle $4GM/Rc^2$ in the weak-field, static limit. The shear-softening coefficient $\alpha = -2$ follows directly from the first-order expansion of elastic moduli under pre-stress, constrained only by consistency with the observed deflection; it corresponds to a specific linear combination of third-order Murnaghan elastic constants. This constitutes an existence proof: at least one class of effective material media reproduces this key prediction of General Relativity. Consequently, gravitational lensing observations alone do not uniquely discriminate between spacetime curvature and anisotropic optical propagation in a mechanically responding medium. This work forms the foundational layer of the Triple-Point Vacuum (TPV) program; the same refractive-index profile is shown in companion papers to account for the Shapiro time delay [11] and Lorentz invariance [12]. The analysis is restricted to the weak-field, static regime.

Keywords: Gravitational Lensing; Analog Gravity; Elastic Vacuum; Continuum Mechanics; Acousto-Elasticity; General Relativity; Emergent Gravity; Triple-Point Vacuum

1 Introduction

Gravitational lensing has played a central role in the empirical validation of General Relativity (GR). Early Newtonian corpuscular theories predicted that light would be deflected by massive bodies, but yielded a deflection angle $\delta = 2GM/Rc^2$—only half the value $4GM/Rc^2$ later derived from GR and confirmed by Eddington's 1919 observations [1]. This discrepancy is commonly cited as evidence that gravity cannot be described by any scalar or isotropic refractive mechanism, and that the tensorial structure of spacetime curvature is essential.

However, the Newtonian refractive models that yielded the factor-of-two discrepancy all shared a key restriction: they treated the effective medium as a scalar fluid, varying only in density. Modern continuum mechanics admits a richer material response. In a stressed elastic solid, wave speeds depend not only on density but also on the local state of stress through the *acousto-elastic effect* [4]—a well-established laboratory phenomenon in which applied stresses modify elastic moduli and hence wave speeds. When a medium is subject to a radially symmetric compression field, transverse and longitudinal modes respond differently, producing an anisotropic refractive index that a scalar fluid model cannot capture.

This paper addresses the historical factor-of-two objection directly. By expanding to a two-parameter acousto-elastic model we recover the full GR deflection angle. The result is presented strictly as a *proof of concept*—an existence proof that the objection is not fatal to medium-based interpretations. This work forms the foundational contribution of the Triple-Point Vacuum (TPV) program [13], which models the vacuum as a hyper-elastic continuum admitting three energy-propagation regimes. Companion papers show that the same continuum framework accounts for the Shapiro time delay [11], Lorentz invariance as a dynamic enforcement condition [12], and anomalous galactic rotation curves without dark matter [13].

2 Elastic Wave Propagation in a Continuum

We consider a homogeneous, isotropic elastic continuum characterized by mass density ρ and shear modulus μ. In the linear elastic regime, transverse (shear) waves propagate with speed

$$v = \sqrt{\frac{\mu}{\rho}}. \tag{1}$$

We define a refractive index relative to the asymptotic wave speed $c_0 = \sqrt{\mu_0/\rho_0}$ as

$$n(\mathbf{x}) = \frac{c_0}{v(\mathbf{x})} = \sqrt{\frac{\rho(\mathbf{x})}{\rho_0} \cdot \frac{\mu_0}{\mu(\mathbf{x})}}. \tag{2}$$

This expression separates two independent physical contributions: variations in inertia (density) and variations in restoring force (shear stiffness). In a general stressed medium, both may vary with position and both must be retained for a complete description.

3 Weak-Field Gravitational Compression

3.1 Effective Density Scaling

The Newtonian gravitational potential is

$$\Phi(r) = -\frac{GM}{r}. \tag{3}$$

Letting $\varphi \equiv GM/(rc^2) \ll 1$, mass-energy equivalence implies that a gravitational potential contributes to the effective inertial mass density of any medium coupled to gravity. To first order,

$$\frac{\rho(r)}{\rho_0} \approx 1 + 2\varphi, \tag{4}$$

where the coefficient 2 matches the post-Newtonian isotropic-coordinate representation of the Schwarzschild metric [3].

3.2 Shear Modulus Variation via the Acousto-Elastic Effect

For a hyperelastic material with strain-energy density $W(I_1, I_2, I_3)$, the incremental moduli governing wave speeds in a pre-stressed body differ from their unstressed values by amounts proportional to the third-order (Murnaghan) elastic constants [4, 5]. Under a radially symmetric gravitational compression, the general first-order correction to the shear modulus is

$$\frac{\mu(r)}{\mu_0} \approx 1 + \alpha\,\varphi, \tag{5}$$

where α is a dimensionless acousto-elastic coefficient determined by the Murnaghan constants of the medium. Conventional materials under hydrostatic compression stiffen ($\alpha > 0$); materials with anomalous nonlinear constitutive response can soften ($\alpha < 0$). In the TPV framework the sign of α is regime-dependent: softening near compressed mass concentrations (lensing regime) and mild stiffening in energy-depleted regions relevant to galactic rotation curves [13].

3.3 Derivation of the Acousto-Elastic Coefficient

Substituting Eqs. (4) and (5) into Eq. (2), and noting that $\mu_0/\mu \approx 1 - \alpha\varphi$ to first order,

$$\begin{aligned} n(r) &= \sqrt{\frac{\rho}{\rho_0} \cdot \frac{\mu_0}{\mu}} = \sqrt{(1+2\varphi)(1-\alpha\varphi)} \\ &\approx \sqrt{1+(2-\alpha)\varphi} \\ &\approx 1 + \frac{2-\alpha}{2}\varphi + O(\varphi^2). \end{aligned} \tag{6}$$

The GR-equivalent refractive index in isotropic coordinates is $n_{\mathrm{GR}}(r) = 1 + 2\varphi$ [3]. Matching to Eq. (6) requires

$$\frac{2-\alpha}{2} = 2 \quad \Longrightarrow \quad \boxed{\alpha = -2.} \tag{7}$$

The constitutive relations consistent with GR lensing are therefore

$$\frac{\rho(r)}{\rho_0} = 1 + 2\varphi, \qquad \frac{\mu(r)}{\mu_0} = 1 - 2\varphi, \tag{8}$$

yielding

$$\boxed{n(r) \approx 1 + \frac{2GM}{rc^2}.} \tag{9}$$

Physically, $\alpha = -2$ corresponds to a medium whose shear modulus softens under compression at exactly the same fractional rate as its density increases, but with opposite sign. In terms of Murnaghan's third-order constants (l, m, n_M) [5], the coefficient α is related to the combination $(l + 2m)/\mu_0$ evaluated in the limit of small pre-strain; the TPV value $\alpha = -2$ specifies a corresponding constraint on this combination for the effective vacuum constitutive law.

4 Light Deflection

In geometric optics, the deflection angle δ for a ray passing a mass M at impact parameter R is

$$\delta = \int_{-\infty}^{\infty} \nabla_\perp n \, dx, \tag{10}$$

where x is the coordinate along the unperturbed ray and $\nabla_\perp$ is the gradient perpendicular to it (see Fig. 1).

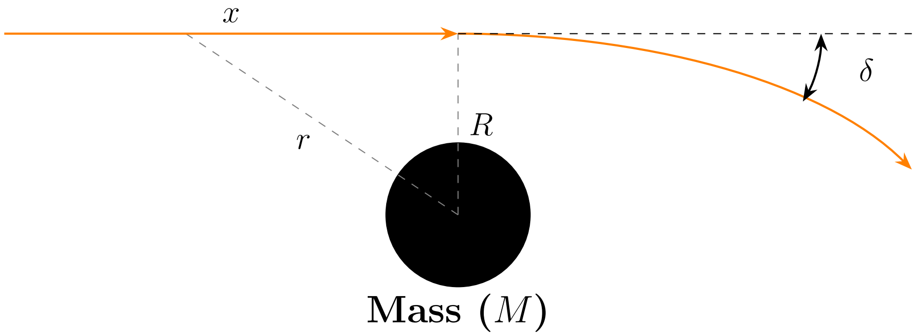

Figure 1: Geometry of gravitational deflection. A ray passes mass M at impact parameter R, acquiring total deflection δ.

Using $n(r)$ from Eq. (9) with $r = \sqrt{x^2 + R^2}$,

$$\nabla_\perp n = \frac{2GM}{c^2} \frac{R}{(x^2 + R^2)^{3/2}}. \tag{11}$$

Evaluating the standard integral,

$$\delta = \frac{2GMR}{c^2} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{2GM}{c^2} \cdot \frac{2}{R} = \boxed{\frac{4GM}{c^2 R}.} \tag{12}$$

This is the weak-field general-relativistic deflection angle.

5 Consistency with Other Weak-Field Tests

The refractive index of Eq. (9) serves as the foundational quantity for two further consistency checks within the TPV program. A companion paper [11] demonstrates that the same index profile reproduces the Shapiro radar time delay [2] to leading logarithmic order in isotropic coordinates. A second companion [12] shows that Lorentz invariance emerges as a dynamic stability condition for wave-based matter structures propagating in the same continuum. Together, these three results establish that a single refractive-index profile unifies the principal weak-field observational signatures of GR within the hyper-elastic continuum framework.

6 Longitudinal Modes and Physical Interpretation

Any elastic continuum supporting transverse shear waves must also support longitudinal (compressional) waves at speed $v_L = \sqrt{(\lambda + 2\mu)/\rho}$, where λ is the first Lamé parameter. Their status in the effective vacuum continuum is the most significant open question for models of this type.

Several possible resolutions have been discussed in the analog-gravity literature: (i) the longitudinal mode may be massive (gapped), excited only at energies far above those of electromagnetic radiation [7]; (ii) longitudinal modes may correspond to scalar gravitational degrees of freedom of Brans–Dicke type, with coupling already constrained by Solar System tests; or (iii) the effective medium description may break down at scales where longitudinal modes would appear, signaling the need for a more complete microphysical theory. Resolution (i) receives partial support from the Lorentz invariance analysis of Ref. [12], which models massive particles as self-stabilized standing-wave structures in the same continuum; if longitudinal modes are similarly localized, they need not propagate freely. A first-principles resolution—deriving the full mode spectrum from vacuum thermodynamics—is in preparation as part of the TPV program.

7 Discussion

7.1 What This Result Demonstrates

The calculation above establishes a precise existence claim: there is at least one class of effective acousto-elastic continua—with density scaling $\rho \propto 1 + 2\varphi$ and shear softening $\mu \propto 1 - 2\varphi$ under gravitational compression—that reproduces the full GR light-deflection angle in the weak-field limit. The acousto-elastic coefficient $\alpha = -2$ is fixed by consistency with lensing observations and is not a free parameter; it specifies a constraint on the third-order Murnaghan constants of the effective vacuum constitutive law. A first-principles derivation of these constitutive relations from vacuum entropy arguments—explaining *why* ρ and μ scale with gravitational potential in precisely this way—is the subject of forthcoming work within the TPV program.

7.2 Regime Asymmetry: Compression vs. Depletion

It is worth noting that the acousto-elastic softening described here ($\alpha = -2$) applies specifically to the compressed vacuum regime near massive bodies. The TPV framework also admits a complementary depletion regime, relevant to galactic scales far from baryonic mass concentrations, in which the vacuum exhibits mild *stiffening* ($\mu_{\text{eff}} > \mu_0$) [13]. The two regimes are physically distinct and consistent: compression and depletion represent opposite departures from the vacuum ground state, with opposite constitutive responses. Gravitational lensing near compact objects probes the compressed regime; anomalous rotation curves probe the depleted regime.

7.3 Relation to Analog Gravity and Observational Discriminants

This work aligns with the analog gravity program, in which effective metrics arise from material properties rather than being fundamental [8, 7]. In the weak-field regime, the present model and GR make identical predictions for the deflection angle and the Shapiro delay.

Distinguishing them requires precision measurements of secondary effects. One natural discriminant is polarimetric: stress-induced optical anisotropy may produce weak birefringence, yielding polarization-dependent phase shifts correlated with the lensing geometry [10]—an effect absent in GR. High-sensitivity multi-band polarimetric observations of strongly lensed systems represent the most direct avenue for falsification or constraint of the elastic continuum interpretation.

7.4 Summary of Predictions

Observable	**GR**	**Elastic Model**	**Status**
Light deflection (weak field)	$4GM/Rc^2$	$4GM/Rc^2$	Match
Shapiro delay	$(2GM/c^3)\ln(4r_1r_2/R^2)$	$(2GM/c^3)\ln(4r_1r_2/R^2)$	Match [11]
Lorentz invariance	Postulate	Dynamic enforcement	Match [12]
Polarization rotation	None	Possible (birefringence)	Discriminant
Perihelion precession	$6\pi GM/ac^2(1-e^2)$	Not yet derived	Future work
Gravitational wave speed	c	Depends on v_L	Open

Table 1: Comparison of key observational predictions between GR and the elastic continuum model.

8 Regime of Validity and Limitations

The analysis applies only to:

- Weak gravitational fields ($GM/rc^2 \ll 1$)
- Static, spherically symmetric sources

- Classical geometric-optics propagation (wavelength $\ll$ curvature scale)
- Transverse wave modes

The model does not address: strong-field effects; dynamical or time-varying sources; gravitational wave generation; frame-dragging; perihelion precession; or a fully covariant field-equation formulation. These define the boundary of the current proof of concept and the agenda for future work within the TPV program.

9 Conclusion

By modeling the vacuum as an effective hyper-elastic continuum with density increase and shear softening ($\alpha = -2$) under gravitational compression, the full general-relativistic light-deflection angle $4GM/c^2R$ emerges naturally in the weak-field limit. The acousto-elastic coefficient follows from the correct first-order expansion of the refractive index and is fixed uniquely by consistency with observations. This resolves the long-standing factor-of-two objection: the failure of historical refractive models arose from restricting attention to density-only (scalar-fluid) media. When anisotropic elastic response is included, the discrepancy vanishes.

The same refractive-index profile is shown in companion papers to reproduce the Shapiro time delay [11] and to yield Lorentz invariance as a dynamic stability condition [12], establishing three-way weak-field consistency of the hyper-elastic continuum program. We do not claim that the vacuum is literally an elastic solid, nor that GR is incorrect. We demonstrate that the effective material description remains viable after the most commonly cited empirical objection is properly accounted for, and that a single index profile unifies three distinct weak-field predictions. This supports the broader Triple-Point Vacuum program and identifies clear targets for future theoretical development and observational test.

Declarations

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Conflict of interest

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Availability of data and materials

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Code availability

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